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On the buckling and the Strouhal law of fluid columns: the case of turbulent jets and wakes

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Abstract

An extension of the buckling model of oscillating fluid columns is presented that considers the nonuniform velocity profiles observed in various free turbulent flows. The buckling Strouhal number, defined as the reciprocal of the dimensionless wavelength, is calculated for some flows well documented in the literature. In all cases, the buckling Strouhal number is the same, despite the difference in flow configurations. The buckling model is compared with the universal Strouhal law, showing that the two models are equivalent to each other. Finally, a relationship between the maximum entropy generation and the difference between predicted and experimental Strouhal number is presented.

1. Introduction

It is well known that all free flows at high Reynolds numbers, behave as if they were nonviscous fluid flows. What is not sufficiently known, is that in this condition, the flow generally looks like a *sinuous* current. Examples in the industrial, aircraft and hydraulic world, are abundant, as referred to by Bejan (1982). Outstanding examples in nature are the meandering of many rivers worldwide (Leopold and Wolman, 1960), the Gulf Stream in the Atlantic ocean, e.g., Lugt (1983) and the melt currents in glaciers. In all cases, it is remarkable that the characteristic wavelength of the oscillations is proportional to the width scale of the given current.

There are many instances in Fluid Mechanics, on the other hand, where repetitive phenomena, almost periodic, are present with clearly distinctive frequencies (Berger and Wille, 1972). This is the case of the lateral oscillations in free turbulent jets and wakes, whose characteristic frequencies, when scaled with the appropriate local variables, compose a unique dimensionless parameter for this class of flows (Cervantes and Goldschmidt, 1981; Bejan, 1981; Levi, 1983).

The origin of these oscillations is the instability of the flow, either at the first stages of the viscous regime, or once the turbulent regime has been established, when the flow impels and drags the still fluid of the environment that it penetrates, experiencing a kind of buckling that manifests itself like almost periodic lateral oscillations. In the first case—the laminar flow, where the oscillations are easily detectable and quantifiable by means of various experimental techniques—the conventional analysis of the problem is the one derived from the theory of linear stability of the flow. The existence of a defined disturbance is postulated, superimposed to a known laminar flow, and its evolution is

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Nomenclature

A	cross-section area
e	oscillations energy
C	impulsive force
D	cross-section flow thickness scale
E	elasticity modulus
F	velocity profile integral, Eq. (10)
f	local frequency of oscillations
$f(\eta)$	similarity shape profiles
I	area moment of inertia
M	bending momentum
R_∞	radius of curvature
$\bar{S}_{gen}$	dimensionless entropy generation rate
$\bar{S}_{gen,max}$	maximum dimensionless entropy generation rate
St	Strouhal number
U	velocity scale
u	longitudinal flow velocity
u_c	center-line velocity
$\bar{u}_c$	dimensionless center-line velocity
V	uniform velocity in a constant cross-section flow
x	longitudinal coordinate
$\bar{x}$	dimensionless longitudinal coordinate
y	transversal coordinate
Y	flexural eccentricity

Greek symbols

η	dimensionless flexural eccentricity
λ	wavelength

Subscripts

B	buckling
L	oscillator

analyzed in time and space. If it grows, the flow is unstable; if on the contrary the disturbance decays, then the flow is considered stable.

The observation and experimental characterization of the oscillations in the second case, that is, in turbulent flows, require the use of specialized instrumentation and techniques of visualization (Bradshaw, 1971; Brown and Roshko, 1974), and of the processing of random signals (Bendat and Piersol, 1971; Cervantes and Goldschmidt, 1981). The results of numerous investigations during the last 30 years impel us to consider that the random characteristics with statistical properties of turbulent flows are true for the small time and space scales of the motion, but not for the large scales (Crown and Champagne, 1971; Davies and Yule, 1975). This fact is specially remarkable in the case of free turbulent flows—jets, wakes and mixing layers—where a coherent and somehow deterministic behavior has been identified (Brown and Roshko, 1974; Davies and Yule, 1975). In many cases, oscillations with clearly distinctive frequencies in the average, have been detected and measured (Cervantes and Goldschmidt, 1981; Levi, 1983). In other words, free turbulent flows (essentially nonviscous), have a regular sinuous aspect.

In this way, it is possible to recognize an almost periodic behavior in turbulent free flows that have been typified in the literature through concepts like coherent structures and combination of vortices in mixing layers and two-dimensional wakes, and the flapping of plane jets. In all these cases, a frequency of oscillation associated to a characteristic wavelength can be quantified, that scales with the flow width and the average local velocity of the fluid, giving rise to a representative Strouhal number for the whole flow. In this paper, a model for the lateral oscillations of free turbulent flows, is presented. It is based on the concept of low-viscosity buckling flows, fully documented in the archival literature (Biot, 1964; Taylor, 1969; Buckmaster et al., 1975; Bejan, 1981).

2. The buckling of flows: an extended model

The phenomenon of buckling has been widely studied with respect to solid columns and other structural elements since it was first introduced by Euler in his pioneering investigations. It consists in a new equilibrium condition that solid structures—columns, plates and vessels—attain under compression loads, characterized by a permanent flexion strain in a more stable general state than that they had before (Shanley, 1965).

The term buckling was employed in Fluid Mechanics for the first time by Biot (1964), studying the large-scale geological structure and the buckling of viscoplastic plates. The first observations and their qualitative discussion were done by Taylor (1969), in respect to vertical falling layers of honey, who mentioned that an analogy could exist between the elastic buckling of a column and the flow of a fluid.

The first theoretical statements about the buckling of a fluid column—called the 'Viscida' theory as an analog to the 'Elastica' theory—is due to Buckmaster et al. (1975), who assumed the absence of inertial effects and considered the action of gravity and compression at the ends of the columnar flow. They concluded that the equation that describes the shape of the fluid column under buckling was similar to the Elastica equation, except that the bending momentum in the first case—the flow column—is proportional to the rate of change of the curvature of the column, whereas the bending momentum for the elastic column is proportional to the curvature itself.

Bejan (1981), extended the buckling concept to a variety of fluid columns, since the viscous flows previously investigated by others represented only a subrange of the full set of possibilities for this concept. Bejan established a buckling theory for flows without *viscidity* (that is, without viscosity, seen as a characteristic of the movement and not as a property of the fluid), that allows the derivation of the scaling laws of the transition to turbulence and the turbulent large scales themselves. The analysis is based on concepts from Mechanics and was formulated for a constant cross-section and uniform velocity in each section of a fluid column. An abridged review is presented in the next paragraphs, essential to support the contributions made by this paper.

Consider the equilibrium conditions for a segment of fluid column as indicated in Fig. 1. The control volume that includes the fluid column simultaneously experiences axial compression and bending moments, as a result of the impulse and reaction forces acting at both extremities.

Translational equilibrium means that there is a balance of the axial compression forces, C; but the rotational equilibrium, from a section where the bending moment is M_0 to any section where the bending moment is $M(x)$, has to be preserved, even when the forces C are not collinear. In this case, the bending moment CY coming out from the eccentricity Y of the compression force, must be balanced at all times by the cross-sectional bending moment $M(x)$, giving rise, therefore, to the infinitesimal Euler buckling

$$-M(x) + CY + M_0 = 0. \tag{1}$$

The analysis of a linear strain (an elongation) associated to pure bending gives, in conclusion, that the bending moment in any section of the column is proportional to the curvature of the column; so in the case of an elastic column, it is

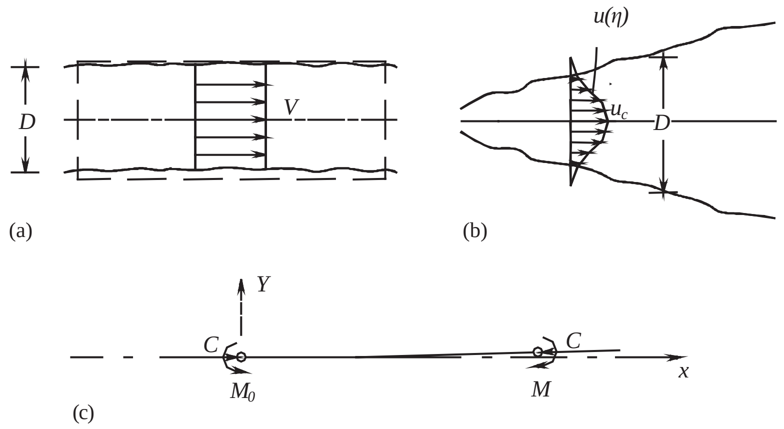

Fig. 1. Translational and rotational equilibrium in a fluid column [adapted from Bejan (1982)]: (a) uniform velocity profile; (b) nonuniform velocity profile.

written as

$$M(x) = \frac{EI}{R_\infty}, \tag{2}$$

where E is the elasticity modulus, I is the area moment of inertia of the cross-section, and R is the radius of curvature, given respectively as

$$I = \int_A y^2\, \mathrm{d}A \quad \text{and} \quad \frac{1}{R_\infty} = -\frac{\mathrm{d}^2 Y}{\mathrm{d}x^2}. \tag{3}$$

Bejan (1981, 1982, 1987) analyzed the case of a fluid column with constant cross-section and uniform velocity distribution in each section, with the assumption that in order to have a curvature for the column, a balance is needed for the forces and accelerations in the transverse direction (that is, between centrifugal forces associated to the curvature and a given lateral distribution for the pressure). If a more detailed analysis is conducted on the linear strain associated to any fiber in pure bending, it can be found that this assumption is not needed.

With these considerations Bejan obtained

$$(\rho V^2 I)Y'' + (\rho V^2 A)Y + M_0 = 0, \tag{4}$$

which has the form

$$Y'' + mY = \text{const.} \tag{5}$$

the prime denotes differentiation with respect to x.

The solution to this equation is represented by a sine function with a single wavelength, as distinct from the buckling of an elastic solid column, where an infinite set of modes that correspond to the eigenvalues of the problem. This is due to the fact that the compressive force and the elasticity modulus are independent of each other, whereas in the case of the fluid column, the terms that correspond to these concepts—the coefficients of the first two terms of Eq. (4)—are proportional each other. The wavelength obtained is

$$\lambda_B = 2\pi\sqrt{\frac{I}{A}}. \tag{6}$$

In the following paragraphs, an extension of the above analysis by Bejan, is presented, based on the fundamental theory of buckling (Shanley, 1965). The analysis uses the same concepts and principles, but it is more general as it can be applied to various cases of turbulent free flows (flows without viscidity), taking into account their nonuniform velocity profiles in a cross-section of the fluid column, as reported in the archival literature.

Considering a self-similar nonuniform velocity profile for any cross-section of the flow column, on a unit depth basis [see for instance, Townsend (1976)], we can write

$$u = u_c f(\eta) \quad \text{with} \quad \eta = Y/D \quad \text{and} \quad \mathrm{d}\eta = \mathrm{d}Y/D, \tag{7}$$

where u_c is the center-line velocity of the flow, and D is the flow thickness scale at every cross-section, Fig. 2.

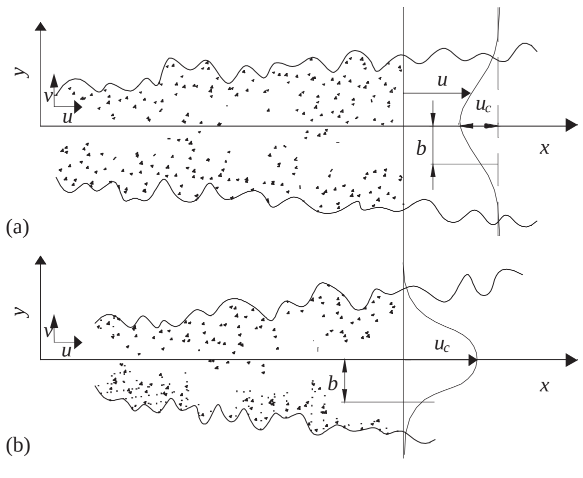

Fig. 2. Schematic diagram of plane free turbulent flows: (a) wake; (b) jet.

The impulsive forces (reactions) at the ends of the column, acting as the analog of an axial compressive load to the fluid column of Fig. 1, are then

$$C = \rho u_c^2 D\left[\int_{-\infty}^{\infty} f^2\,\mathrm{d}\eta\right] \tag{8}$$

and the equilibrium condition given in Eq. (1) can be expressed as

$$\rho u_c^2 D^3\left[\int_{-\infty}^{\infty} f^2\eta^2\,\mathrm{d}\eta\right]\frac{\mathrm{d}^2 Y}{\mathrm{d}x^2} + \rho u_c^2 D\left[\int_{-\infty}^{\infty} f^2\,\mathrm{d}\eta\right] Y + M_0 = 0. \tag{9}$$

Writing F_1 and F_2 for the integrals of the velocity profile in any cross-section of the flow

$$F_1 = \int_{-\infty}^{\infty} f^2\eta^2\,\mathrm{d}\eta \quad \text{and} \quad F_2 = \int_{-\infty}^{\infty} f^2\,\mathrm{d}\eta \tag{10}$$

and dividing all the terms in Eq. (9) by the coefficient of the derivative in the first term, we obtain

$$\frac{\mathrm{d}^2 Y}{\mathrm{d}x^2} + \frac{F_2}{F_1 D^2} Y + M_0' = 0. \tag{11}$$

Note that Eq. (11) has the same form as Eqs. (4) and (5), except for the coefficient of the second term, which is now expressed through the integrals given in Eqs. (10). The single wavelength of the solution of Eq. (11) for this general case is

$$\lambda_\mathrm{B} = 2\pi\sqrt{\frac{F_1 D^2}{F_2}}. \tag{12}$$

For the case where the fluid velocity profile is uniform (the so-called "top-hat" profile), $f(\eta) = 1$, and in Eq. (9) $F_2 D$ reduces to the cross-section area, and $F_1 D^3$ to the moment of inertia of this area I; so, Eq. (12) simplifies to Eq. (6)

$$\lambda_B = 2\pi\sqrt{\frac{I}{A}},$$

i.e., the result obtained by Bejan is recovered.

With these theoretical considerations, the dimensionless wavelengths of turbulent jets and wakes that have an oscillatory behavior, were calculated in this investigation, using Eq. (12), with velocity profiles amply documented in the literature. Table 1 summarizes these results and includes, at the rightest column, the *buckling Strouhal number*, defined as the reciprocal of the dimensionless wavelength

$$\mathrm{St}_B = \frac{D}{\lambda_B}. \tag{13}$$

As can be seen in Table 1, an interesting result is that almost the same value is obtained for the calculated dimensionless wavelengths for all the considered flow cases, despite that they correspond to different flow

Table 1
Velocity profiles, dimensionless wavelengths and Strouhal numbers, for various free turbulent flows

Flow description	Velocity profile	Velocity profile integrals Eq. (10)		Dimensionless wavelength Eq. (12)	Buckling Strouhal number Eq. (13)
		F_1	F_2	$\frac{\lambda_B}{D} = 2\pi\sqrt{\frac{F_1}{F_2}}$	$\mathrm{St}_B = \frac{D}{\lambda_B}$
Plane wake	$f(\eta) = \exp\left(-\frac{1}{2}\eta^2\right)$	0.0719	0.922	1.83	0.546
Plane jet (top-hat)	$f(\eta) = 1$	0.0833	1.0	1.81	0.552
Plane jet (hyperbolic tangent)	$f(\eta) = 1 - \tanh^2(\eta)$	0.0627	0.858	1.69	0.591
Plane jet (hyperbolic secant)	$f(\eta) = \mathrm{sec\,h}^2\left[(2/\pi)^{1/2}\eta\right]$	0.0693	0.904	1.74	0.575
Round jet	$f(\eta) = \left(1 + \frac{\eta^2}{2}\right)^{-2}$	0.0629	0.859	1.7	0.588

configurations. They confirm the order of magnitude estimate of 0.5 obtained by Bejan (1982) and agree with the experimental measurements of the natural frequency of turbulent jets exposed to a range of external excitation frequencies, as reported by the same author. He considered this value as an universal constant. The slight differences among them are due to the different degree of spreading of the corresponding profile, giving rise to different values of the integrals in Eq. (10).

3. The Strouhal law

It has been reported by Levi (1983) that many examples of periodic flow phenomena, even with little or no resemblance among them, can be represented by the same value of the dimensionless wavelength, or the reciprocal *Strouhal number*, if the proper local or representative variables are considered. Levi summarized a detailed review of a large collection of oscillatory flow processes with frequency f, that arise when a given flow current of mean velocity U, interacts with a still body of fluid, of characteristic length dimension D. These included wakes, jets, shock waves, cavitation, wing autorotation, vortex breakdowns and surface and internal waves. Experimental values of fD/U around 0.159 were claimed by this author for such a variety of flow situations. The Strouhal law proposed by Levi (1983), is an attempt to offer a simple explanation for the periodic oscillations (frequency f) in a restrained fluid body in direct contact with a free current flow (velocity U). He assumed that if the free current induces the layer (of width D) to vibrate, then this can be modelled as an elementary oscillator (actually a resonator) with characteristic vibration energy given by

$$e = \tfrac{1}{2}(2\pi D f)^2, \tag{14}$$

while the available specific kinetic energy of the current flow is given by $U^2/2$. By equating them, he obtained

$$\frac{fD}{U} = \frac{1}{2\pi} = 0.159,$$

which defines in a simple way the *universal Strouhal law*, as Levi coined it

$$\mathrm{St}_L = \frac{fD}{U} = 0.159. \tag{15}$$

Levi completes his paper by giving a summary of his findings after a literature search for several flow cases, documenting *only* those cases where experimental results were published. These are briefly listed in Table 2, where the Strouhal numbers remarkably are well clustered around the value predicted by Eq. (15).

A full explanation of the rationale for a common Strouhal number in such diverse flow situations is still lacking. Nevertheless, its universality has also been recognized in the case of jets, wakes and similar free columnar flows, by contributors like Bejan (1982, p.77).

Table 2
Summary of the Strouhal numbers for different flow situations, according to Levi (1983)

Flow description	St_L
Wakes behind cylinders (Roshko, 1954; Richter and Naudascher, 1976; Griffin, 1978)	0.164
Flapping turbulent plane jets (Cervantes and Goldschmidt, 1981)	0.154
Orderly structure in turbulent round jets (Crown and Champagne, 1971)	0.15
Bursting in geophysical flows (Jackson, 1976)	0.16
Vortices behind weirs (Levi, 1979)	0.154
Periodical vortex structures (Levi, 1980)	0.176
Autorotating wings (Smith, 1971)	0.16
Flow over cavities (Rossiter, 1964)	0.16
Von Karman vortex street (Goldstein, 1965)	0.16
Wake axisymmetric airfoil (Sato and Okada, 1966)	0.151

4. Discussion

The two models for the oscillations of free flows reviewed and extended in this paper, are equivalent to each other. The difference between Levi's Strouhal number ($St_L = 0.159$), and Bejan's Strouhal number ($St_B \approx 0.5$), is only because of the manner that this dimensionless parameter is defined, i.e., dividing or not by π. In other words, $St_B = \pi St_L$.

The buckling model by Bejan comes out from a momentum conservation analysis, while the resonator model by Levi is the result of an energy conservation analysis. Both consider a conservative system, thus they offer the same result. The buckling model can be viewed as a distributed parameter representation in the sense that part of the geometrical and kinematic properties of the flow column are taken into account through the velocity profile shape for each flow. For this reason there are slight differences in the calculated value of the Strouhal number, as mentioned before. The resonator model on the other hand, corresponds to a lumped parameter description, giving therefore a single value of the Strouhal number.

Whichever is the case, both models are the limit of the real behavior of free turbulent flows. The evolution of any real flow always goes accompanied by entropy generation which, in the case of a free flow, intuitively increases as a function of entrainment at the flow boundaries, as well as of mixing and lateral extension of the flow with the surrounding, still fluid. Bejan (1996) has recently developed the theoretical basis to quantify the entropy generation in a variety of physical situations of interest.

In a recent paper (Cervantes and Solorio, 2002), the entropy generation for an oscillating turbulent plane jet, was calculated. Their results corresponded to the physically expected behavior. Turbulence represents the natural tendency of a flow field to seek and find a flow structure that enhances the mixing rate. The participation of the oscillations in the flow gives as a result that part of the flow energy that must contribute to the entrainment with the still ambient fluid and mixing process of turbulence. Higher values for entropy generation were obtained in each jet cross-section due to the lateral oscillations, as compared to the calculated entropy generation for the case without oscillations. The smooth increase of the production of entropy in the downstream direction, towards an asymptotic maximum value, confirmed the similarity properties of the flow in the far-field region.

The analysis can be extended to other free turbulent flows, like a plane wake and a round jet, to obtain a response similar to the one described in that paper for plane jets. Table 3 and Fig. 3 show these results. In Table 3 the buckling Strouhal number according to Eq. (13), is listed together with the experimental values referred to by Levi (1983) and the maximum entropy generation calculated after Cervantes and Solorio (2002). As noted, there is a difference between the calculated and the reported experimental Strouhal numbers (third column of Table 3). This difference—small for the wake and larger for the round jet, with the plane jet in between—takes higher values in correspondence to the raised values of the maximum entropy generation.

This maximum value of the entropy generation is attained at large values of the downstream coordinate and is used to normalize the entropy generation growth along the direction of the flow. This is plotted in Fig. 3. As seen, there is the same trend for entropy generation in the three flows, with the plane wake having the smaller maximum value and thus attaining a higher normalized trend, in a shorter downstream distance. A similarity behavior can also be observed. According to Townsend (1976), self-preservation may exist only as an asymptotic condition, asserting that a moving equilibrium is setup in which the conditions at the initiation of the flow are largely irrelevant, and the flow depends on a few simple parameters and is geometrically similar at all sections. This seems to be the case with respect to entropy generation in buckling or oscillating free turbulent flows.

Table 3
Maximum entropy generation for some free turbulent flows, including the effect of oscillations through the Strouhal number

Flow description	Strouhal number[a] St_B	Strouhal number[b] $\pi\, St_L$	Strouhal number difference	$\bar{S}_{gen,max}$[c]
Plane wake	0.546	0.51	0.036	17.43
Plane jet (hyperbolic tangent)	0.591	0.48	0.111	18.59
Round jet	0.588	0.47	0.118	19.27

[a] Eq. (13) of the present paper.
[b] Experimental values as referred to by Levi (1983).
[c] Calculated according to Cervantes and Solorio (2002).

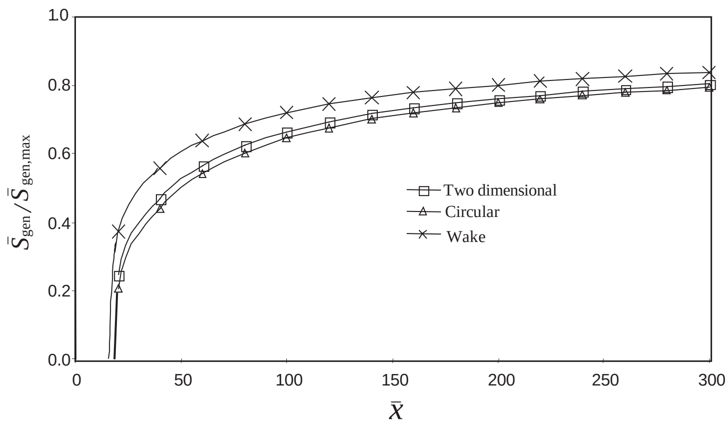

Fig. 3. Normalized entropy generation for a plane jet, a plane wake and a round jet, with oscillations, calculated after Cervantes and Solorio (2002).

5. Concluding remarks

An extension of the buckling theory of flows developed by Bejan (1981), has been presented for turbulent oscillating free flows. The extended model considers nonuniform velocity profiles and includes the simplest case of uniform velocity analyzed by Bejan. This theory was confronted with a parallel model proposed by Levi (1983), showing that both theses agree with each other and predict the same Strouhal number. Any difference in their predictions with the values calculated in the present paper, and with experimental results reported in the literature, could be explained as a result of the entropy generation that is coupled with any real flow. The smooth increase of the production of entropy in the downstream direction, towards an asymptotic maximum value, that resulted from the analysis, confirms the similarity properties of the flow in the far field region for basic turbulent free flows. The buckling modelling of oscillating turbulent flows and the entropy generation that accompanies it are difficult to characterize and need further work. Nevertheless, their main results presented herein may contribute to a better understanding of the buckling interpretation and have not previously been reported in the literature.

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